Logic I: Fast Lecture 02

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Formal Proof: ∧Elim and ∧Intro

Reading: §5.1, §6.1

Conjunction Introduction
(∧ Intro)

P_1
$\Downarrow$
P_n
$\vdots$
▷ $P_1 \wedge \ldots \wedge P_n$

Conjunction Elimination
(∧ Elim)

$P_1 \wedge \ldots \wedge P_i \wedge \ldots \wedge P_n$
$\vdots$
▷ P_i

1.	P ∧ Q	
2.	Q ∧ R	
3.	P	∧Elim: 1
4.	R	∧Elim: 2
5.	P ∧ R	∧Intro: 3,4

2. awFOL symbol words

symbol	word(s)
¬	not
→	arrow, ->
↔	double_arrow, <->
⊥	false, contradiction
∧	and, &
∨	or, ||
↓	nor
↑	nand
∀	all, every
∃	some, exists

3. ∧Intro and ∨Intro: Compare and Contrast

Reading: §6.1

Disjunction Introduction
(∨ Intro)

P_i
$\vdots$
▷ $P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n$

1.	R	
2.	S	
3.	R ∨ T	∨Intro: 1
4.	S ∧ (R ∨ T)	∧Intro: 2,3

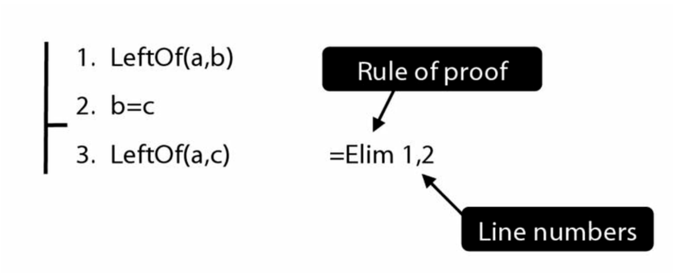
Let us define a new connective with this truth table:

P1	P2	P1 ∨ P2	P1 $\not\leftrightarrow$ P2
T	T	T	F
T	F	T	T
F	T	T	T
F	F	F	F

The following rule is unacceptable. Why?

$\not\leftrightarrow$Intro:
Pi
...
P1 $\not\leftrightarrow$ P2

4. How to Write Proofs



5. Rules of Proof for Identity

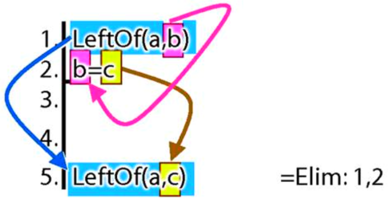
Reading: §2.2

Identity Introduction (= Intro)

▷ | n = n

Identity Elimination (= Elim)

| P(n)
| ⋮
| n = m
| ⋮
▷ | P(m)



6. DeMorgan: $\neg(A \wedge B) \dashv\vDash \neg A \vee \neg B$

Reading: §3.6

'$\dashv\vDash$' means 'is logically equivalent to', so for now 'has the same truth table as'.

$A \dashv\vDash \neg\neg A$

$\neg(A \wedge B) \dashv\vDash (\neg A \vee \neg B)$

$\neg(A \vee B) \dashv\vDash (\neg A \wedge \neg B)$

$A \longrightarrow B \dashv\vDash \neg A \vee B$

$\neg(A \longrightarrow B) \dashv\vDash \neg(\neg A \vee B) \dashv\vDash A \wedge \neg B$

7. →Intro, →Elim

Reading: §8.1, §8.2

Conditional Introduction (→ Intro)

| | P
| | ⋮
| | Q
▷ | P → Q

Conditional Elimination (→ Elim)

| P → Q
| ⋮
| P
| ⋮
▷ | Q

8. →Intro: An Example

1. P→Q
2. Q→R

6. P→R

9. ∨Intro and ∨Elim

Disjunction Introduction (∨ Intro)

P_i
⋮
▷ $P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n$

Disjunction Elimination (∨ Elim)

$P_1 \vee \ldots \vee P_n$
⋮
P_1
⋮
S

⇓

P_n
⋮
S
⋮
▷ S

10. ∨Elim: An Example

To prove a conclusion from a disjunction, prove it from each disjunct.

1. R∨S

2.
3.

4.
5.

6. S∨R

11. ¬, ⊥

Reading: §6.3

P	¬P	⊥
T	F	F
F	T	F

⊥ Introduction (⊥ Intro)

P
⋮
¬P
⋮
▷ ⊥

⊥ Elimination (⊥ Elim)

⊥
⋮
▷ P

12. ¬Elim

Reading: §6.3

Negation Elimination
(¬ Elim)

¬¬P
⋮
▷ P

13. Scope: A Mistaken Application of ¬Elim

What is wrong with this proof?

1.	¬¬(¬A ∧¬¬ A)	
2.	(¬A ∧ ¬¬A)	¬Elim: 1
3.	(¬A ∧ A)	¬Elim: 2

14. ¬Intro

Reading: §5.3, §6.3

Negation Introduction
(¬ Intro)

P
⋮
⊥
▷ ¬P