Logic I: Fast Lecture 03

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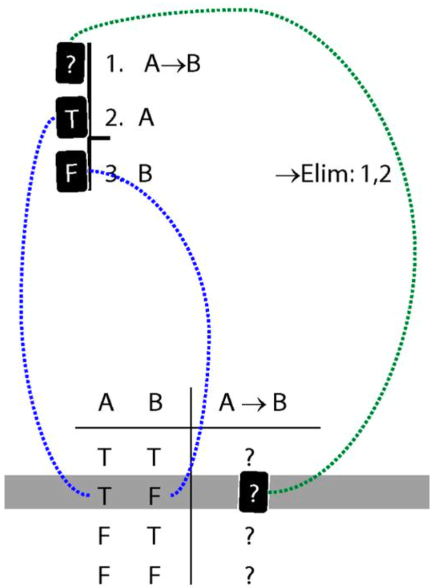
Readings refer to sections of the course textbook, *Language, Proof and Logic.*

1. What does '→' mean?

Reading: §7.1

Assuming that the rules of Fitch are such that it is impossible to prove an argument which is not logically valid, the truth table for → is fixed if we accept →Elim and →Intro.

How do the rules of proof for → fix its truth table?



2. ¬Elim

Reading: §6.3

Negation Elimination (¬ Elim)

| ¬¬P
| ⋮
▷ | P

3. Scope: A Mistaken Application of ¬Elim

What is wrong with this proof?

1. $\neg\neg(\neg A \land \neg\neg A)$	
2. $(\neg A \land \neg\neg A)$	¬Elim: 1
3. $(\neg A \land A)$	¬Elim: 2

4. ¬Intro

Reading: §5.3, §6.3

Negation Introduction (¬ Intro)

| | P
| | ⋮
| | ⊥
▷ | ¬P

1. $A \land B$

$\neg(\neg A \lor \neg B)$

5. $A \wedge B \vee C$

Reading: §3.5

Ambiguity can be *lexical*, e.g. ‘Actor testifies in horse suit’. Ambiguity can also be *syntactic*, e.g. ‘How to combat the feeling of helplessness with illegal drugs’. (Both examples are from Bucaria, C. (2004), ‘Lexical and syntactic ambiguity as a source of humor: The case of newspaper headlines’, Humour 17(3): 279–309.)

6. $A \wedge B \vee C$: They Are Different

Argument 1

1. $(A \wedge B) \vee C$
2. $A \wedge (B \vee C)$

Argument 2

1. $A \wedge (B \vee C)$
2. $(A \wedge B) \vee C$

7. I Shot an Elephant in My Pyjamas

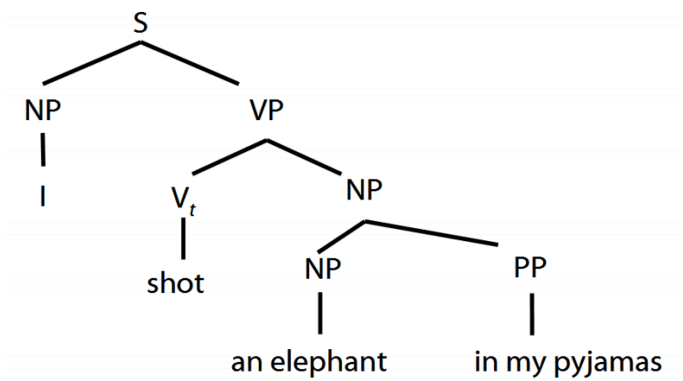
Rule 1: a NP followed by a VP is a S

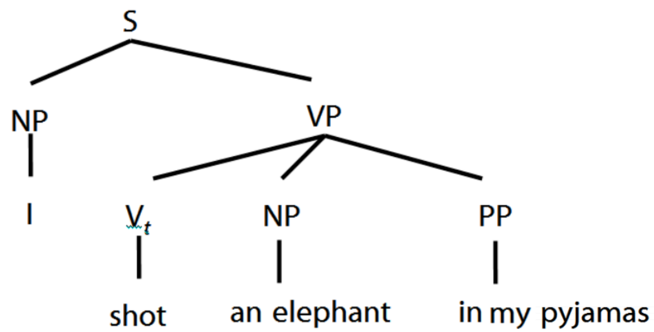
Rule 2: a Vt followed by a NP is a VP

Rule 3: a NP followed by a PP is a S

Rule 4: A Vt followed by a NP then a PP is a VP

Two derivations of Groucho Marx’ claim, ‘I shot an elephant in my pyjamas’:





8. The Syntax of awFOL

Reading: §9.3

We define what counts as a sentence of awFOL using rules. E.g.:

1. If * and # are sentences, then so is(* ∧ #)
2. If * and # are sentences, then so is (* ∨ #)
3. P, Q, R, … are sentences
4. If * is a sentence, then ¬* is a sentence

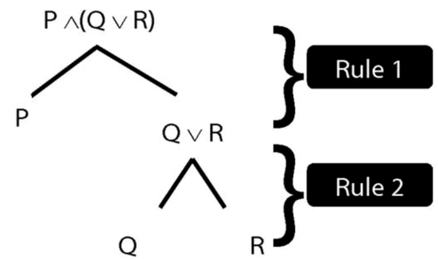
So:

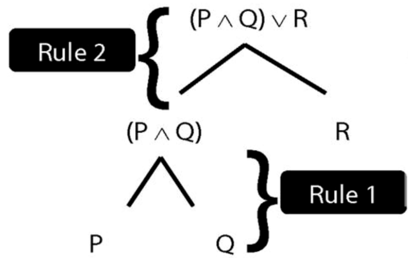
a. P is a sentence // rule 3

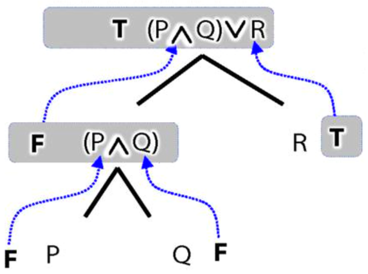
b. ¬P is a sentence // rule 4, a

c. (¬P ∧ Q) is a sentence // rule 1, b, a

There is no structural ambiguity in awFOL because these rules are formulated to ensure that for any awFOL sentence, there is exactly one way of constructing it.







9. Scope: A Mistaken Application of ¬Elim

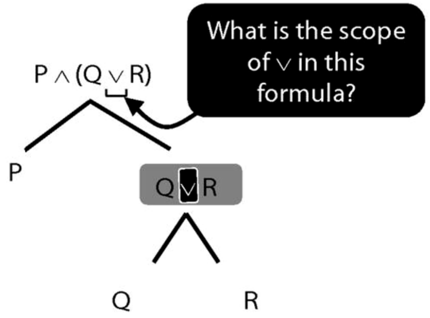
What is wrong with this proof?

1.	$\neg\neg(\neg A \wedge \neg\neg A)$	
2.	$(\neg A \wedge \neg\neg A)$	¬Elim: 1
3.	$(\neg A \wedge A)$	¬Elim: 2

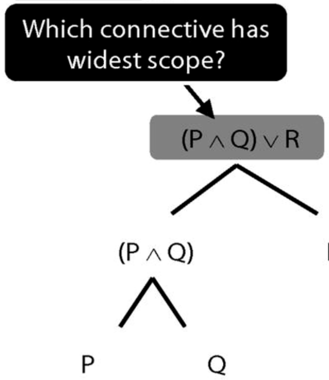
10. Scope

Reading: §3.5

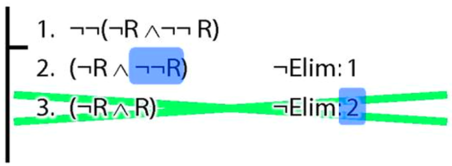
The *scope* of a connective (token) is the sentence containing it lowest in the tree.



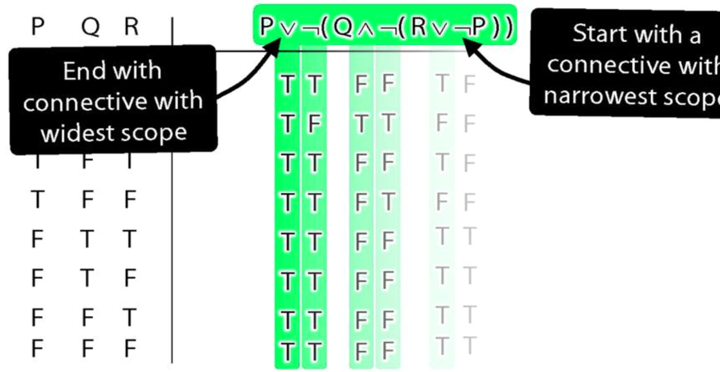
The connective with *widest scope* is the one whose scope is the whole sentence.



A rule of proof can only be applied to the connective with widest scope.



When we do truth tables, the order we do the columns in is determined by scope.



11. Truth-functional Connectives

Reading: §7.0 (the text before §7.1)

A *connective* joins zero or more sentences to make a new sentence. Examples of connectives include: ‘∧’, ‘¬’, ‘⊥’ and ‘because’.

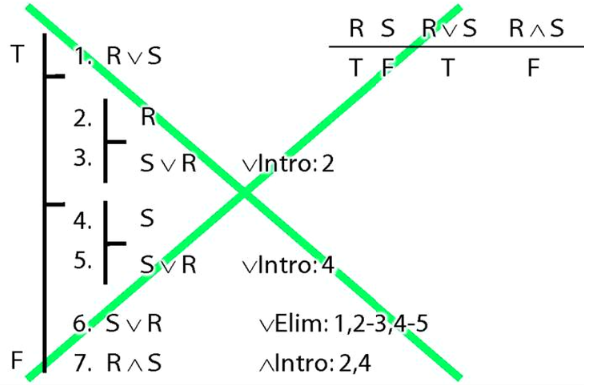
A sentence joined by a connective is a *constituent*. For example, consider the sentence ‘P because Q’: P is a constituent of this sentence.

A *truth functional connective* produces a new sentence whose truth value depends only on the truth values of its constituent sentences.

When P and Q are both true, ‘P because Q’ is sometimes true and sometimes false. Therefore, ‘because’ is not a truth functional connective. To illustrate, consider ‘Alan got yellow cards because some apples are green' and 'Alan got yellow cards because he used his elbows'. All the constituent sentences are true, but the first sentence is false whereas the second is true.

12. Subproofs Are Tricky

What is wrong with the following apparent proof?



13. Everything Is Broken

Reading: §9.1, §9.2

Everything is broken: ∀x Broken(x)

Something is broken: ∃x Broken(x)