Logic I: Fast Lecture 04

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Everything Is Broken

Reading: §9.1, §9.2

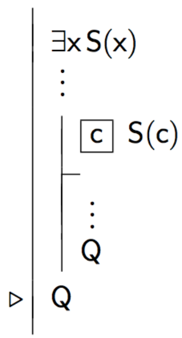
Everything is broken: ∀x Broken(x)

Something is broken: ∃x Broken(x)

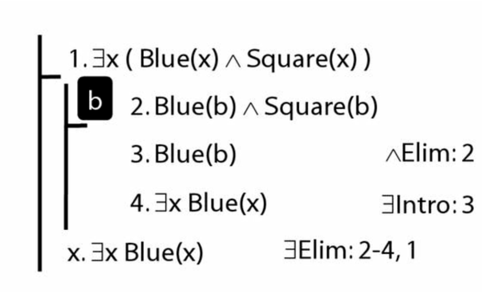
2. ∃Elim

Reading: §12.2, §13.2

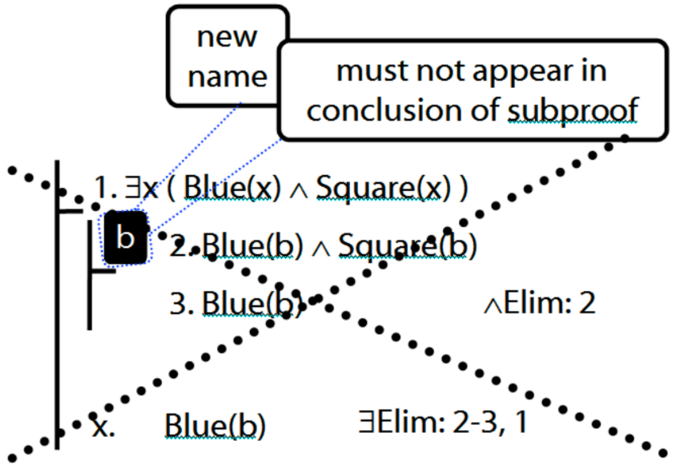
Existential Elimination (∃ Elim)



where c does not occur outside the subproof where it is introduced.



Note this restriction on the use of ∃Elim:



3. All Squares Are Blue (Fast Version)

Reading: §9.2, §9.3, §9.5

∃ and ∧ work together
Some square is blue:
∃x (Square(x) ∧ Blue(x))
Some of my things are broken:
∃x (Belongs(a,x) ∧ Broken(x))

∀ and ⟶ work together
All squares are blue:
∀x (Square(x) ⟶ Blue(x))
All my things are broken:
∀x (Belongs(a,x) ⟶ Broken(x))

4. What does ∀ mean?

Reading: §9.4

We give the meaning of ∀ by specifying what it takes for a sentence containing ∀ to be true:

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name.
3. If ALL of the new sentences are true, so is the original sentence.

5. ∀Intro

Reading: §12.1, §12.3, §13.1

Universal Introduction
(∀ Intro)

$\boxed{c}$

⋮

P(c)

▷ ∀x P(x)

where c does not occur outside the subproof where it is introduced.

1.	∀x (Square(x) → Blue(x))	
2.	∀x Square(x)	
3.	a	
4.	Square(a)	∀Elim: 2
5.	Square(a)→Blue(a)	∀Elim: 1
6.	Blue(a)	→Elim: 4,5
x.	∀x Blue(x)	∀Intro: 3-6
y.	∀x Square(x) → ∀x Blue(x))	→Intro: 2-x

Why is this proof incorrect?

1.	∀x Square(x) → ∀x Blue(x)	
2.	b	
3.	Square(b) → Blue(g)	∀Elim: 1
x.	∀x (Square(x) → Blue(x))	∀Intro: 2-3

6. Summary of Quantifier Rules

Reading: §13.1, §13.2

∀Elim
If it's true of everything it's true of Baudrillard

∃Intro
If it's true of Baudrillard it's true of something

∃Elim
If it's true of something and Q follows no matter which something it is, then Q

∀Intro
If it's true of an arbitrary thing, then it's true of everything.

7. Scope and Quantifiers

Reading: §9.5, §9.6

Underlining shows the scope of the quantifiers:

"All squares are blue"
$\underline{\mathbf{\forall x\ (\ Square(x) \rightarrow Blue(x)\ )}}$

"If everything is square, everything is blue"
$\underline{\mathbf{\forall x\ Square(x)}}\ \mathbf{\rightarrow}\ \mathbf{\forall x\ Blue(x)}$

8. Translation with Quantifiers

Reading: §9.5, §9.6

All discordians weep:
∀x(Dscrdn(x) ⟶ Wps(x))

All **French** discordians weep:
∀x((**Frnch(x)** ∧ Dscrdn(x)) ⟶ Wps(x))

All French discordians weep **and wail**:
∀x((Frnch(x) ∧ Dscrdn(x)) ⟶ (Wps(x) ∧ **Wls(x)**))

All French discordians weep and wail **except Gillian Deleude**:
∀x((Frnch(x) ∧ Dscrdn(x) ∧ ¬**(x=a)**) ⟶ (Wps(x) ∧ Wls(x)))

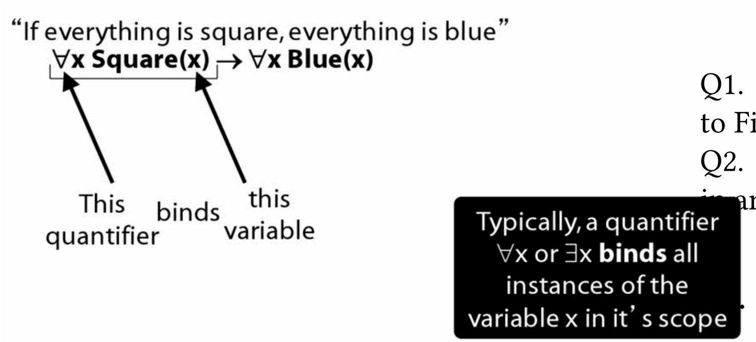
9. What does ∃ mean?

We give the meaning of ∃ by specifying what it takes for a sentence containing ∃ to be true:

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name.
3. If ANY of the new sentences are true, so is the original sentence.

10. Quantifiers Bind Variables

Reading: §9.3



11. Substitution of Equivalents

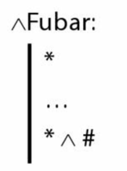
Reading: §4.5, §10.3

Suppose that φ, ψ and χ are sentences of FOL. Suppose that φ is logically equivalent to ψ. Let $\chi[\varphi/\psi]$ be the result of replacing, in χ, zero or more occurrences of φ with ψ. The *subsitution theorem* says that $\chi[\varphi/\psi]$ is logically equivalent to χ.

12. Fubar Rules

Reading: §8.3

Consider this made-up rule:



Q1. What would be wrong with adding ∧Fubar to Fitch?
Q2. What would be wrong with having ∧Fubar in any system of proof?

Something Is Above Something

Reading: §11.1

Something is above something:
$\exists x\, \exists y\, Above(x,y)$

14. Two Things Are Broken

Reading: §14.1

To translate sentences involving number into FOL, use identity. For example,

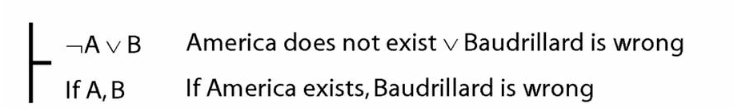
'Two things are broken' might be translated as:

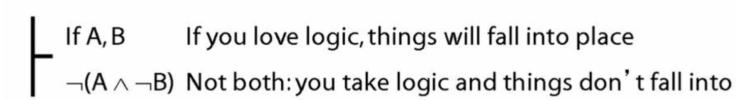
$\exists x\, \exists y\, (\, Broken(x) \wedge Broken(y) \wedge \neg(x=y)\,)$

15. Does 'if' mean what '→' means?

Reading: §7.3

These two arguments are valid: does that mean that 'if' means what '→' means?





The English argument isn't valid; the FOL argument is valid; therefore 'if' can't mean what '→' means?