Logic I: Fast Lecture 05

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Readings refer to sections of the course textbook, *Language, Proof and Logic.*

1. Vegetarians Are Evil

Reading: §9.2, §9.3, §9.5

$\forall x\ (\text{Evil}(x) \rightarrow \text{HatesMeat}(x))$

$\forall x\ (\text{HatesMeat}(x) \rightarrow \text{Vegetarian}(x))$

$\forall x\ (\text{Vegetarian}(x) \rightarrow \text{Evil}(x))$

2. Counterexamples with Quantifiers

	Evil(x)	HatesMeat(x)	Vegetarian(x)
Ayesha	no	no	yes

3. Something Is Above Something

Reading: §11.1

Something is above something:
$\exists x\ \exists y\ \text{Above}(x,y)$

4. Multiple Quantifiers: Everyone Likes Puffins

Reading: §11.1

I like puffins:

$\forall x\ (\text{Puffin}(x) \rightarrow \text{Likes}(a,x))$

y likes puffins:

$\forall x\ (\text{Puffin}(x) \rightarrow \text{Likes}(y,x))$

Everyone likes puffins:

$\forall y\ \forall x\ (\text{Puffin}(x) \rightarrow \text{Likes}(y,x))$

5. Relations: Transitivity

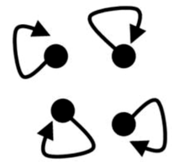
Reading: §15.1

A *transitive* relation is one such that if x bears it to y and y bears it to z then x bears it to z. (E.g. LeftOf is transitive; NotAdjacent is not transitive.)

6. Expressing Relations with Quantifiers

Reading: §15.1

A *reflexive* relation is one that everything bears to itself. (E.g. SameShape)
reflexive: $\forall x\ R(x,x)$



A *symmetric* relation is one such that if x bears it to y, then y bears it to x. (E.g. Adjacent(x,y))
symmetric: $\forall x \forall y\ (R(x,y) \rightarrow R(y,x))$

A *transitive* relation is one such that if x bears it to y and y bears it to z then x bears it to z. (E.g. LeftOf is transitive; DifferentShape is not transitive)
transitive: $\forall x \forall y \forall z\ ((R(x,y) \land R(y,z)) \rightarrow R(x,z))$

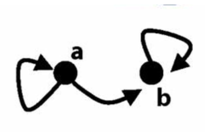
7. Expressing Counterexamples Formally

Reading: §15.1

Give a counterexample to this argument:

∀x R(x,x)
∀x∀y∀z ([R(x,y) ∧ R(y,z)] → R(x,z))
———
∀x∀y [R(x,y) → R(y,x)]

Informally:



Formally:

Domain: {a, b}

R: {<a,a>, <a,b>, <b,b>}

8. There Is a Store for Everything

Reading: §11.2, §11.3

There is a store for everything:

∃y∀x StoreFor(y,x)

∀y∃x StoreFor(x,y)

Other sentences to translate:

Wikipedia has an article about everything

Everyone hurts someone they love

Someone hurts everyone she loves

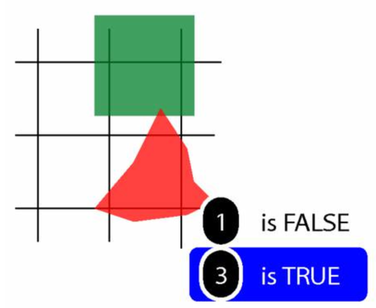
9. ∀Intro: An Incorrect Proof

Reading: §13.1, §13.2

This proof is wrong, but why?:

(3) 1. ∀x Square(x) → ∀x Blue(x)
  2. [b]
  3. Square(b) → Blue(b)   ∀Elim: 1
(1) x. ∀x (Square(x) → Blue(x))   ∀Intro: 2-3

There is a counterexample to the argument:



10. Two Things Are Broken

Reading: §14.1

To translate sentences involving number into FOL, use identity. For example,

'Two things are broken' might be translated as:

∃x ∃y (Broken(x) ∧ Broken(y) ∧ ¬(x=y))

11. Quantifier Equivalences: ∀x(Square(x) ⟶ Broken(x)) ⫤⊨ ∀x(¬Broken(x) ⟶ ¬Square(x))

Reading: §10.3

P	Q	P→Q	¬Q→¬P
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

12. Quantifier Equivalences: ∀x Created(x) ⫤⊨ ¬∃x Created(x)

Reading: §10.3, §10.4

13. Soundness and Completeness: Statement of the Theorems

Reading: §8.3, §13.4

'A $\vdash$ B' means there is a proof of B using premises A

'$\vdash$ B' means there is a proof of B using no premises

'A $\vDash$ B' means B is a logical consequence of A

'$\vDash$ B' means B is a tautology

'A $\vDash_{TT}$ B' means B is a logical consequence of A just in virtue of the meanings of truth-functions (the textbook LPL calls this 'tautological consequence')

Soundness: If A $\vdash$ B then A $\vDash$ B

i.e. if you can prove it in Fitch, it's valid

Completeness: If A $\vDash_{TT}$ B then A $\vdash$ B

i.e. if it's valid just in virtue of the meanings of the truth-functional connectives, then you can prove it in Fitch.