

Logic I: Lecture 03

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Logical Validity and Truth Tables

Reading: §4.3

Truth tables can be used to show that an argument is valid. To illustrate ...

P	Q	$P \vee Q$	$\neg P$	Q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

\wedge \wedge \wedge
 premise premise conclusion

P	Q	$P \vee Q$
T	T	T
F	T	T
T	F	T
F	F	F

$\neg P$
 Q

To establish that an argument is valid:

1. Create truth tables for each premise and the conclusion.
2. Check whether there is a row of the truth table where all premises are true and the conclusion is false.
3. If not, the argument is valid.

2. Formal Proof: \wedge Elim and \wedge Intro

Reading: §5.1, §6.1

Conjunction Introduction (\wedge Intro)

P_1
\downarrow
P_n
\vdots
$\triangleright P_1 \wedge \dots \wedge P_n$

Conjunction Elimination (\wedge Elim)

$P_1 \wedge \dots \wedge P_i \wedge \dots \wedge P_n$
\vdots
$\triangleright P_i$

1. $P \wedge Q$	
2. $Q \wedge R$	
3. P	\wedge Elim: 1
4. R	\wedge Elim: 2
5. $P \wedge R$	\wedge Intro: 3,4

Writing a proof in logic-ex

premises (2)

A \wedge B
B \wedge C

conclusion

A \wedge C

Your answer:

1	A \wedge B	
2	B \wedge C	
3	---	
4	A	// and elim 1
5	C	// and elim 2
6	A \wedge C	// and intro 4,5

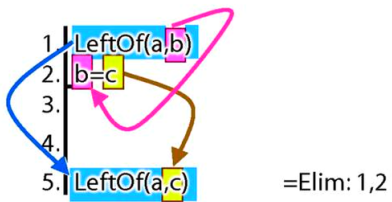
3. Rules of Proof for Identity

Reading: §2.2

Identity Introduction (= Intro)

$\triangleright n = n$

Identity Elimination (= Elim)

$$\begin{array}{|l} P(n) \\ \vdots \\ n = m \\ \vdots \\ P(m) \end{array}$$


P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Annotations:

- Always start with T (points to top row)
- Sentence letters are ordered alphabetically (points to columns P, Q, R)
- Right-most column alternates every row (points to R column)
- Always end with F (points to bottom row)
- Next right-most column alternates half as often as previous column (points to Q column)
- Next right-most column alternates half as often (points to P column)

6. Logic Makes Me Die Inside

Reading: §2.1

7. Contradictions, Logical Truths and Logical Validity

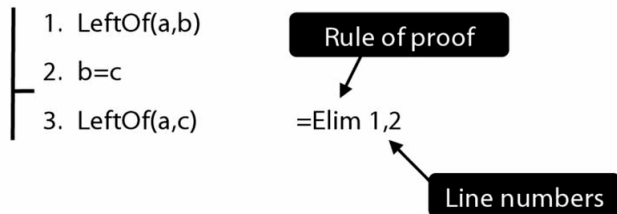
Reading: §4.1, §4.2, §5.4

Argument 3
 \vdash 1. $(P \wedge Q) \vee R$
 2. $P \vee \neg P$

Argument 3b
 \vdash 1. $P \vee \neg P$

Argument 4
 \vdash 1. $P \wedge \neg P$
 2. $(P \wedge Q) \vee R$

4. How to Write Proofs



Complex truth table example:

P	Q	R	$(P \wedge Q) \vee R$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

$P \vee \neg P$ is a *logical truth*
 logical truth defined p. 568
 $P \wedge \neg P$ is a *contradiction*
 contradiction defined p. 564

5. Complex Truth Tables

Reading: §3.3, §3.5