

Logic I: Lecture 2

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Why Logic?

'Logic pervades the world: the limits of the world are also its limits.' (Wittgenstein, *Tractatus* 5.61)

'If a card has a vowel on one side, then it has an even number on the other side.' (Wason & Johnson-Laird 1972)



2. Recap: Validity, Counterexamples

An argument is *logically valid* just if there's no possible situation in which the premises are true and the conclusion false

A *counterexample* to an argument is a possible situation in which its premises are T and its conclusion F.

3. Logical Validity and Truth Tables

Reading: §4.3

Truth tables can be used to show that an argument is valid. To illustrate ...

| P | Q | $P \vee Q$ | $\neg P$ | Q |
|---|---|------------|----------|---|
| T | T | T | F | T |
| T | F | T | F | F |
| F | T | T | T | T |
| F | F | F | T | F |

\wedge \wedge \wedge
 premise premise conclusion

| P | $P \vee Q$ |
|---|------------|
| T | T |
| F | F |

$\neg P$
 Q

| P | Q | R |
|---|---|---|
| T | T | T |
| T | T | F |
| T | F | T |
| T | F | F |
| F | T | T |
| F | T | F |
| F | F | T |
| F | F | F |

Always start with T

Sentence letters are ordered alphabetically

Right-most column alternates every row

Always end with F

Next right-most column alternates half as often

Next right-most column alternates half as often as previous column

To establish that an argument is valid:

1. Create truth tables for each premise and the conclusion.
2. Check whether there is a row of the truth table where all premises are true and the conclusion is false.
3. If not, the argument is valid.

4. Complex Truth Tables

Reading: §3.3, §3.5

Complex truth table example:

| P | Q | R | $(P \wedge Q) \vee R$ |
|---|---|---|-----------------------|
| T | T | T | T |
| T | T | F | T |
| T | F | T | T |
| T | F | F | F |
| F | T | T | T |
| F | T | F | F |
| F | F | T | T |
| F | F | F | F |

5. Tautologies and Contradictions

Reading: §4.1, §4.2

Argument 3
 \vdash 1. $(P \wedge Q) \vee R$
 \vdash 2. $P \vee \neg P$

Argument 3b
 \vdash 1. $P \vee \neg P$

Argument 4
 \vdash 1. $P \wedge \neg P$
 \vdash 2. $(P \wedge Q) \vee R$

$P \vee \neg P$ is a *logical truth*

logical truth defined p. 568

$P \wedge \neg P$ is a *contradiction*

contradiction defined p. 564

Conjunction Introduction $(\wedge$ Intro)

$$\begin{array}{l} | P_1 \\ | \downarrow \\ | P_n \\ | \vdots \\ \triangleright P_1 \wedge \dots \wedge P_n \end{array}$$

Conjunction Elimination $(\wedge$ Elim)

$$\begin{array}{l} | P_1 \wedge \dots \wedge P_i \wedge \dots \wedge P_n \\ | \vdots \\ \triangleright P_i \end{array}$$

$$\begin{array}{l} | 1. P \wedge Q \\ | 2. Q \wedge R \\ | 3. P \qquad \wedge\text{Elim: 1} \\ | 4. R \qquad \wedge\text{Elim: 2} \\ | 5. P \wedge R \qquad \wedge\text{Intro: 3,4} \end{array}$$

6. Formal Proof: \wedge Elim and \wedge Intro

Reading: §5.1, §6.1