Logic I: Lecture 2

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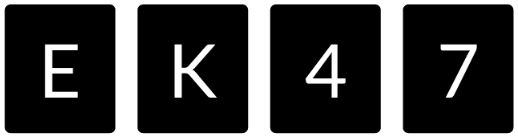
Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Why Logic?

'Logic pervades the world: the limits of the world are also its limits.' (Wittgenstein, Tractatus 5.61)

'If a card has a vowel on one side, then it has an even number on the other side.' (Waison & Johnson-Laird 1972)



2. Recap: Validity, Counterexamples

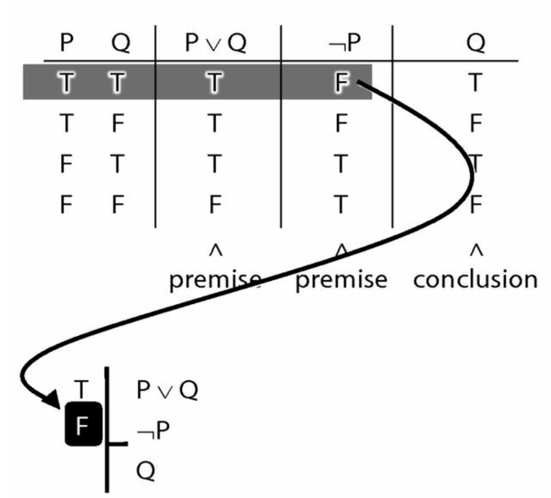
An argument is *logically valid* just if there's no possible situation in which the premises are true and the conclusion false

A *counterexample* to an argument is a possible situation in which its premises are T and its conclusion F.

3. Logical Validity and Truth Tables

Reading: §4.3

Truth tables can be used to show that an argument is valid. To illustrate ...

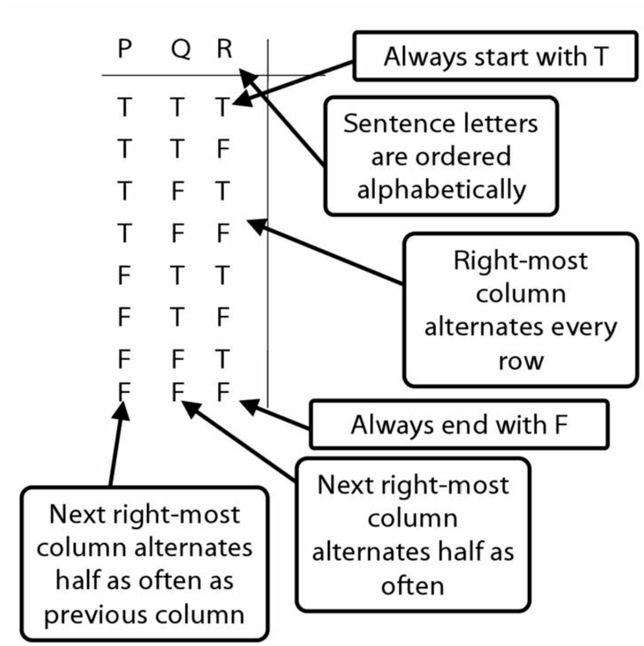


To establish that an argument is valid:

1. Create truth tables for each premise and the conclusion.
2. Check whether there is a row of the truth table where all premises are true and the conclusion is false.
3. If not, the argument is valid.

4. Complex Truth Tables

Reading: §3.3, §3.5



Complex truth table example:

P	Q	R	(P ∧ Q) ∨ R
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

5. Tautologies and Contradictions

Reading: §4.1, §4.2

Argument 3

1. $(P \wedge Q) \vee R$
2. $P \vee \neg P$

Argument 3b

1. $P \vee \neg P$

Argument 4

1. $P \wedge \neg P$
2. $(P \wedge Q) \vee R$

$P \vee \neg P$ is a *logical truth*

logical truth defined p. 568

$P \wedge \neg P$ is a *contradiction*

contradiction defined p. 564

6. Formal Proof: ∧Elim and ∧Intro

Reading: §5.1, §6.1

Conjunction Introduction (∧ Intro)

P_1

⇓

P_n

⋮

▷ $P_1 \wedge \ldots \wedge P_n$

Conjunction Elimination (∧ Elim)

$P_1 \wedge \ldots \wedge P_i \wedge \ldots \wedge P_n$

⋮

▷ P_i

1.	$P \wedge Q$	
2.	$Q \wedge R$	
3.	P	∧Elim: 1
4.	R	∧Elim: 2
5.	$P \wedge R$	∧Intro: 3,4