Logic (PH133): Lecture 4

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Everything Is Broken

Reading: §9.1, §9.2

Everything is broken: ∀x Broken(x)

Something is broken: ∃x Broken(x)

2. What does ∃ mean?

Reading: §9.4

We give the meaning of ∃ by specifying what it takes for a sentence containing ∃ to be true:

1. Give every object a name.

2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name.

3. If ANY of the new sentences are true, so is the original sentence.

3. →Intro, →Elim

Reading: §8.1, §8.2

Conditional Introduction (→ Intro)

Conditional Elimination (→ Elim)

4. ↔ : truth tables and rules

A	B	A ↔ B
T	T	T
T	F	F
F	T	F
F	F	T

Biconditional Elimination (↔ Elim)

Biconditional Introduction (↔ Intro)

5. ∨ **Intro**

Disjunction Introduction
(∨ Intro)

$$\begin{array}{c|l} & \mathsf{P}_i \\ & \vdots \\ \triangleright & \mathsf{P}_1 \vee \ldots \vee \mathsf{P}_i \vee \ldots \vee \mathsf{P}_n \end{array}$$

6. ∨ **Elim: An Example**

To prove a conclusion from a disjunction, prove
it from each disjunct.

$$\begin{array}{ll} 1. & \mathsf{R} \vee \mathsf{S} \\ \\ 2. & \\ 3. & \\ \\ 4. & \\ 5. & \\ \\ 6. & \mathsf{S} \vee \mathsf{R} \end{array}$$

7. ∨ **Elim and Soundness**

Reading: §5.2, §6.2