# Logic (PH133): Lecture 6

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Readings refer to sections of the course textbook, *Language, Proof and Logic.*

## 1. DeMorgan: $\neg(A \wedge B) \dashv\models \neg A \vee \neg B$

*Reading:* §3.6, §4.2

'$\dashv\models$' means 'is logically equivalent to', so for now 'has the same truth table as'.

$A \dashv\models \neg\neg A$

$\neg(A \wedge B) \dashv\models (\neg A \vee \neg B)$

$\neg(A \vee B) \dashv\models (\neg A \wedge \neg B)$

$A \rightarrow B \dashv\models \neg A \vee B$

$\neg(A \rightarrow B) \dashv\models \neg(\neg A \vee B) \dashv\models A \wedge \neg B$

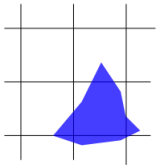
## 2. Negation and the arrow: $A \rightarrow \neg B \not\models \neg(A \rightarrow B)$

*Reading:* §3.6

| A | B | A → ¬B | A → B | ¬(A → B) |
|---|---|---|---|---|
| T | T | F | T | F |
| T | F | T | F | T |
| F | T | T | T | F |
| F | F | T | T | F |

## 3. Don't use ∃ with →

Is true $\exists x(\text{Square}(x) \rightarrow \text{Broken}(x))$ in this world?



$\exists x(\text{Square}(x) \rightarrow \text{Broken}(x))$

$\dashv\models$

$\exists x(\neg\text{Square}(x) \vee \text{Broken}(x))$

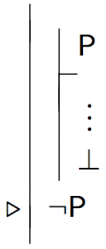
$\dashv\models$

$\exists x(\neg\text{Square}(x)) \vee \exists x(\text{Broken}(x))$

## 4. ¬Intro

*Reading:* §5.3, §6.3

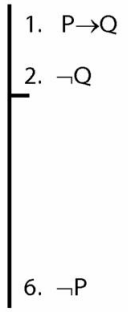
**Negation Introduction (¬ Intro)**





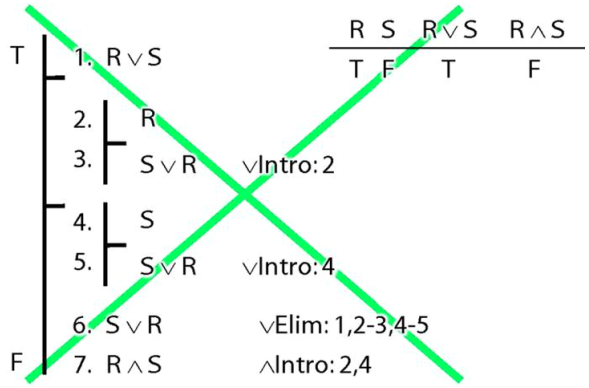
## 5. ¬Intro Proof Example

*Reading:* §5.3, §6.3



## 6. Subproofs Are Tricky

What is wrong with the following apparent proof?




## 7. ∀Elim

*Reading:* §13.1

**Universal Elimination**
**(∀ Elim)**