

Logic (PH133): Lecture 7

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Something Is Above Something

Reading: §11.1

Something is above something:

$$\exists x \exists y \text{ Above}(x,y)$$

2. There Is Exactly One

There is one creator (at least one, maybe more).

$$\exists x \text{ Creator}(x)$$

Ahura Mazda is the one and only creator.

$$\text{Creator}(a) \wedge \forall x (\text{Creator}(x) \rightarrow x=a)$$

All squares are broken.

$$\forall x (\text{Sqr}(x) \rightarrow \text{Brkn}(x))$$

There is one and only one creator.

$$\exists y (\text{Creator}(y) \wedge \forall x (\text{Creator}(x) \rightarrow x=y))$$

or:

$$\exists y \forall x (\text{Creator}(x) \leftrightarrow x=y)$$

3. \exists Intro

Reading: §13.2

Disjunction Introduction (\vee Intro)

$$\begin{array}{|l} P_i \\ \vdots \\ \hline \triangleright P_1 \vee \dots \vee P_i \vee \dots \vee P_n \end{array}$$

4. \exists Elim

Reading: §12.2, §13.2

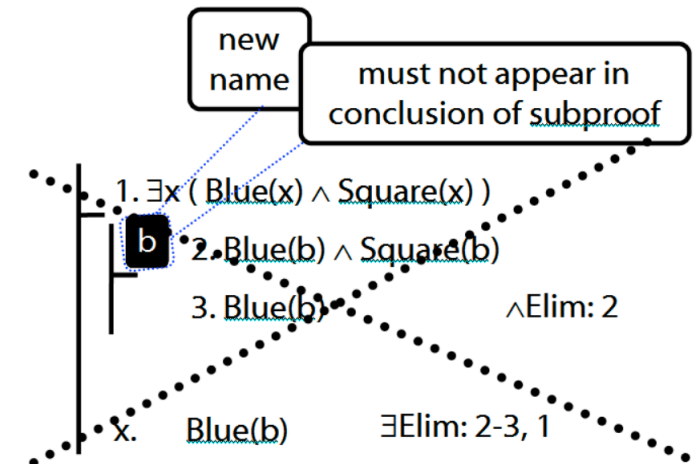
Existential Elimination (\exists Elim)

$$\begin{array}{|l} \exists x S(x) \\ \vdots \\ \hline \boxed{c} S(c) \\ \vdots \\ Q \\ \hline \triangleright Q \end{array}$$

where c does not occur outside the subproof where it is introduced.

$$\begin{array}{|l} 1. \exists x (\text{Blue}(x) \wedge \text{Square}(x)) \\ \hline \boxed{b} 2. \text{Blue}(b) \wedge \text{Square}(b) \\ \hline 3. \text{Blue}(b) \quad \wedge\text{Elim: } 2 \\ \hline 4. \exists x \text{Blue}(x) \quad \exists\text{Intro: } 3 \\ \hline x. \exists x \text{Blue}(x) \quad \exists\text{Elim: } 2-4, 1 \end{array}$$

Note this restriction on the use of \exists Elim:



5. Translation with Quantifiers

Reading: §9.5, §9.6

All discordians weep:

$$\forall x (\text{Dscrdn}(x) \rightarrow \text{Wps}(x))$$

All French discordians weep:

$$\forall x ((\text{Frnch}(x) \wedge \text{Dscrdn}(x)) \rightarrow \text{Wps}(x))$$

All French discordians weep and wail:

$\forall x((\text{Frnch}(x) \wedge \text{Dscrnd}(x)) \rightarrow (\text{Wps}(x) \wedge \text{Wls}(x)))$

All French discordians weep and wail **except Gillian Deleude**:

$\forall x((\text{Frnch}(x) \wedge \text{Dscrnd}(x) \wedge \neg(x=a)) \rightarrow (\text{Wps}(x) \wedge \text{Wls}(x)))$

6. Scope and Quantifiers

Reading: §9.5, §9.6

Underlining shows the scope of the quantifiers:

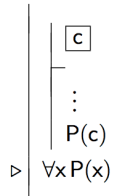
"All squares are blue"
 $\forall x(\underline{\text{Square}(x)} \rightarrow \underline{\text{Blue}(x)})$

"If everything is square, everything is blue"
 $\underline{\forall x \text{Square}(x)} \rightarrow \forall x \underline{\text{Blue}(x)}$

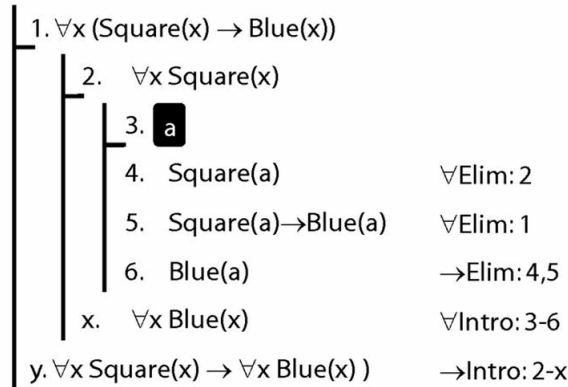
7. \forall Intro

Reading: §12.1, §12.3, §13.1

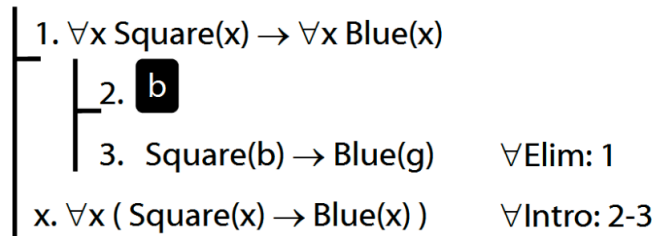
Universal Introduction
 (\forall Intro)



where c does not occur outside the subproof where it is introduced.



Why is this proof incorrect?



8. Summary of Quantifier Rules

Reading: §13.1, §13.2

\forall Elim

If it's true of everything it's true of Baudrillard

\exists Intro

If it's true of Baudrillard it's true of something

\exists Elim

If it's true of something and Q follows no matter which something it is, then Q

\forall Intro

If it's true of an arbitrary thing, then it's true of everything.

9. Two Things Are Broken

Reading: §14.1

To translate sentences involving number into FOL, use identity. For example,

'Two things are broken' might be translated as:

$\exists x \exists y (\text{Broken}(x) \wedge \text{Broken}(y) \wedge \neg(x=y))$