Logic (PH133): Lecture 8

s.butterfill@warwick.ac.uk

Readings refer to sections of the course textbook, *Language, Proof and Logic.*

1. There Is a Store for Everything

Reading: §11.2, §11.3

There is a store for everything:

∃y∀x StoreFor(y,x)

∀y∃x StoreFor(x,y)

Other sentences to translate:

Wikipedia has an article about everything

Everyone hurts someone they love

Someone hurts everyone she loves

2. Variables

Names : a, b, c, ...

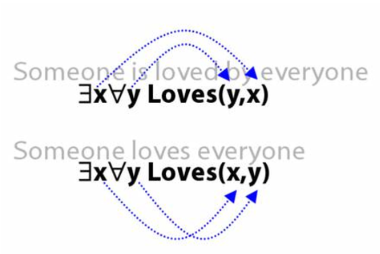
Variables : x, y, z, w, ...

Variables are for saying several things about one thing even without specifying which thing it is

NB: 'Some x is a horse and x is dead' ain't English.

3. Loving and Being Loved

Reading: §11.2, §11.3



4. There Does Not Exist

Something is not dead:

∃x ¬Dead(x)

Nothing is dead:

¬∃x Dead(x)

Everything is not broken:

∀x ¬Broken(x)

Not everything is broken:

¬∀x Broken(x)

1.		
2.	a=a	=Intro
3.	∃x (x=x)	∃Intro: 2

1.	¬∃x Dead(x)	
2.	Dead(a)	
3.	∃x Dead(x)	∃Intro: 2
4.	⊥	⊥Intro: 1,3
5.	¬Dead(a)	¬Intro: 2-4
6.	∃x ¬Dead(x)	∃Intro: 5

1.	∃x ¬Dead(x)
2.	¬∃x Dead(x)

Counterexample:

Domain: {a,b}

Dead : {b}

5. Quantifier Equivalences: ¬∀x Created(x) ⫤⊨ ∃x ¬Created(x)

Reading: §10.1, §10.3, §10.4

6. Proof Example: $\exists x\ Dead(x) \vdash \neg\forall x \neg\ Dead(x)$.

1.	$\exists x\ F(x)$	
2.	a $F(a)$	
3.	$\forall y \neg F(y)$	
4.	$\neg F(a)$	$\forall$Elim: 3
5.	$\perp$	$\perp$Intro: 2,4
6.	$\neg\forall y \neg F(y)$	$\neg$Intro: 3-5
7.	$\neg\forall y \neg F(y)$	$\exists$Elim: 2-6

7. Proof Example: $\neg\forall x\ Dead(x) \vdash \exists x \neg\ Dead(x)$.

1.	$\neg\forall x\ Dead(x)$	
2.	$\neg\exists x \neg Dead(x)$	
3.	a	
4.	$\neg F(a)$	
5.	$\exists x \neg Dead(x)$	$\exists$Intro: 4
6.	$\perp$	$\perp$Intro: 2,5
7.	$\neg\neg F(a)$	$\neg$Intro: 4-6
8.	$F(a)$	$\neg$Elim: 8
9.	$\forall x\ Dead(x)$	$\forall$Intro: 3-8
10.	$\perp$	$\perp$Intro: 1,9
11.	$\neg\neg\exists x \neg Dead(x)$	$\neg$Intro: 2-10
12.	$\exists x \neg Dead(x)$	$\neg$Elim: 11

8. Somebody Is Not Dead

Some person is dead.

$\exists x(Person(x) \wedge Dead(x))$

Some person is not dead.

$\exists x(Person(x) \wedge \neg Dead(x))$

No person is dead.

$\neg\exists x(Person(x) \wedge Dead(x))$

Every person is dead.

$\forall x(Person(x) \longrightarrow Dead(x))$

Every person is not dead.

$\forall x(Person(x) \longrightarrow \neg Dead(x))$

Not every person is dead.

$\neg\forall x(Person(x) \longrightarrow Dead(x))$

9. The End Is Near

Reading: §14.3

'The' can be a quantifier, e.g. 'the square is broken'. How to formalise it?

The square is broken
$\dashv\models$ There is exactly one square and it is broken

Recall that we can translate 'There is exactly one square' as:

$\exists x\ (\ Square(x) \wedge \forall y\ (\ Square(y) \longrightarrow x{=}y\ )\ )$

So 'There is exactly one square and it's broken':

$\exists x\ (\ Sqr(x) \wedge \forall y\ (\ Sqr(y) \longrightarrow x{=}y\ ) \wedge Broken(x)\ )$