Logic I: Lecture 07

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Readings refer to sections of the course textbook, *Language, Proof and Logic.*

1. The Syntax of FOL

Reading: §9.3

We define what counts as a sentence of FOL using rules. E.g.:

1. If * and # are sentences, then so is(* ∧ #)

2. If * and # are sentences, then so is (* ∨ #)

3. P, Q, R, … are sentences

4. If * is a sentence, then ¬* is a sentence

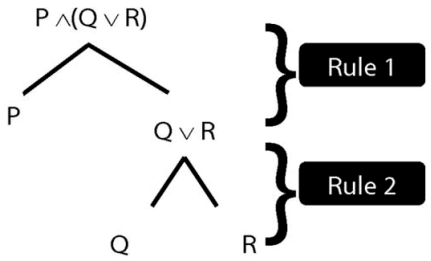
So:

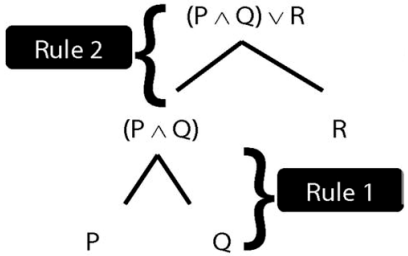
a. P is a sentence // rule 3

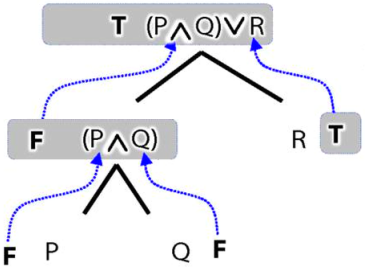
b. ¬P is a sentence // rule 4, a

c. (¬P ∧ Q) is a sentence // rule 1, b, a

There is no structural ambiguity in FOL because these rules are formulated to ensure that for any FOL sentence, there is exactly one way of constructing it.



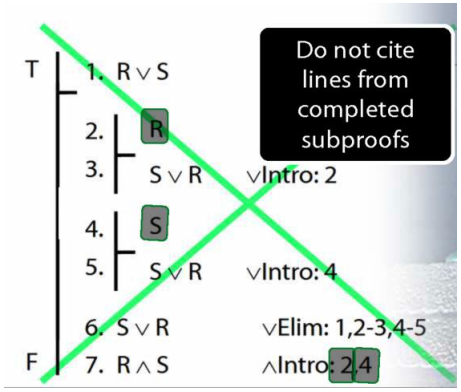




2. ¬P ∨ ¬Q compared with ¬(P ∨ Q)

Reading: §3.5

3. Subproofs Are Tricky: The Answer



4. ¬Intro Proof Example

Reading: §5.3, §6.3

5. DeMorgan: $\neg(A \wedge B) \dashv\models \neg A \vee \neg B$

Reading: §3.6, §4.2

'$\dashv\models$' means 'is logically equivalent to', so for now 'has the same truth table as'.

$A \dashv\models \neg\neg A$

$\neg(A \wedge B) \dashv\models (\neg A \vee \neg B)$

$\neg(A \vee B) \dashv\models (\neg A \wedge \neg B)$

$A \longrightarrow B \dashv\models \neg A \vee B$

$\neg(A \longrightarrow B) \dashv\models \neg(\neg A \vee B) \dashv\models A \wedge \neg B$

6. Everything Is Broken

Reading: §9.1, §9.2

Everything is broken: $\forall x$ Broken(x)

Something is broken: $\exists x$ Broken(x)