Logic I: Lecture 08

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Everything Is Broken

Reading: §9.1, §9.2

Everything is broken: ∀x Broken(x)

Something is broken: ∃x Broken(x)

2. All Squares Are Blue

Reading: §9.2, §9.3, §9.5

3. What does ∀ mean?

Reading: §9.4

We give the meaning of ∀ by specifying what it takes for a sentence containing ∀ to be true:

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name.
3. If ALL of the new sentences are true, so is the original sentence.

4. Vegetarians Are Evil

Reading: §9.2, §9.3, §9.5

∀x (Evil(x) → HatesMeat(x))

∀x (HatesMeat(x) → Vegetarian(x))

∀x (Vegetarian(x) → Evil(x))

5. Not If

If she has seen it, I am dead.
A → B

That's not true.
¬(A → B)

If she has seen it, I am not dead.
A → ¬B

A	B	A → B	¬(A → B)	A → ¬B
T	T	T	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

6. Scope: A Mistaken Application of ¬Elim

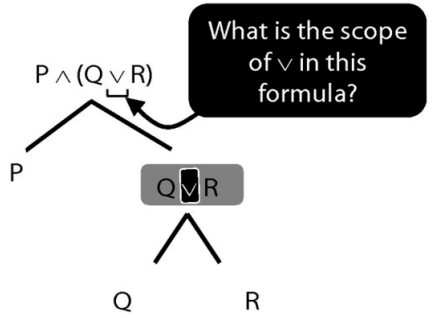
What is wrong with this proof?

1. ¬¬(¬A ∧¬¬ A)
2. (¬A ∧ ¬¬A) ¬Elim: 1
3. (¬A ∧ A) ¬Elim: 2

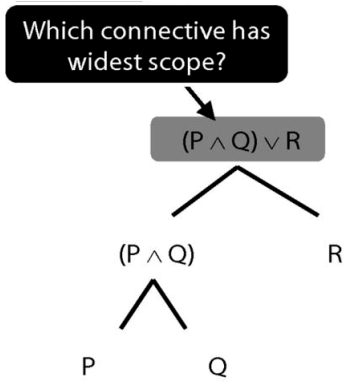
7. Scope

Reading: §3.5

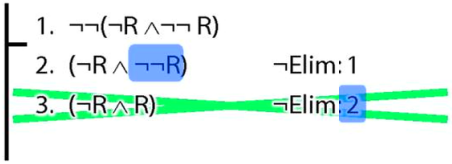
The *scope* of a connective (token) is the sentence containing it lowest in the tree.



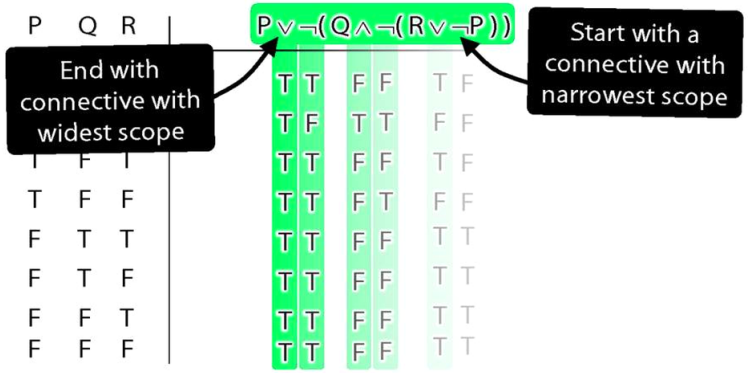
The connective with *widest scope* is the one whose scope is the whole sentence.



A rule of proof can only be applied to the connective with widest scope.



When we do truth tables, the order we do the columns in is determined by scope.



8. Proof Example: ¬(P∨Q) therefore ¬P∧¬Q