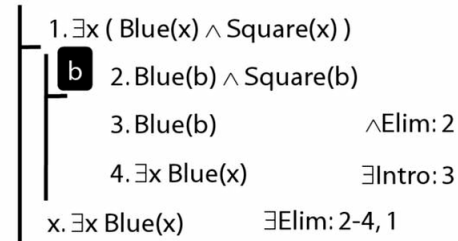# Logic I: Lecture 11

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Revision: ∀Elim, ∃Intro

Reading: §12.1, §13.1, §13.2

2. ∃Elim

Reading: §12.2, §13.2

Existential Elimination (∃ Elim)

$$\begin{array}{|l}
\exists x\, S(x) \\
\vdots \\
\quad \boxed{c}\ S(c) \\
\quad \vdots \\
\quad Q \\
\rhd\ Q
\end{array}$$

where c does not occur outside the subproof where it is introduced.
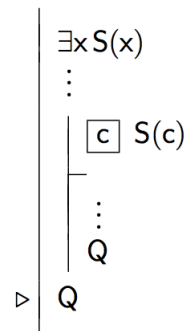
1. ∃x (Blue(x) ∧ Square(x))
     b   2. Blue(b) ∧ Square(b)
        3. Blue(b)   ∧Elim: 2
        4. ∃x Blue(x)   ∃Intro: 3
     x. ∃x Blue(x)   ∃Elim: 2-4, 1

Note this restriction on the use of ∃Elim:

new name — must not appear in conclusion of subproof

1. ∃x (Blue(x) ∧ Square(x))
     b   2. Blue(b) ∧ Square(b)
        3. Blue(b)   ∧Elim: 2
     x.   Blue(b)   ∃Elim: 2-3, 1

3. Don't use ∃ with →

Is true ∃x(Square(x) → Broken(x)) in this world?

∃x(Square(x) → Broken(x))

⊨⊨

∃x(¬Square(x) ∨ Broken(x))

⊨⊨
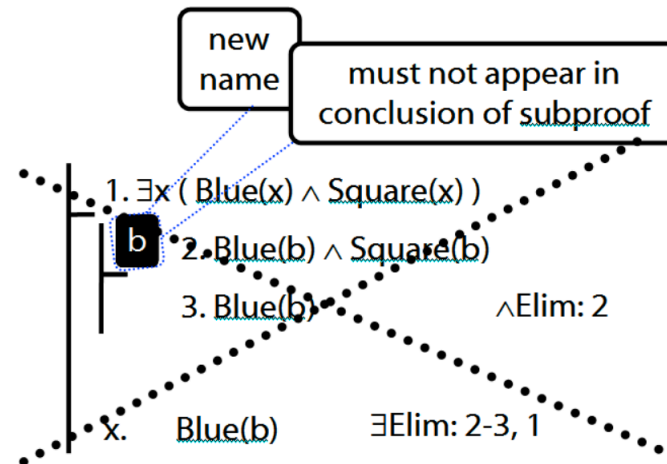
∃x(¬Square(x)) ∨ ∃x(Broken(x))

4. Watch Out, Here Come Multiple Quantifiers
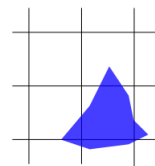
Reading: §11.1

5. Something Is Above Something

Reading: §11.1

Something is above something:
     ∃x ∃y Above(x,y)

6. Multiple Quantifiers: Everyone Likes Puffins

Reading: §11.1

I like puffins:

$\forall x\ (\ Puffin(x) \rightarrow Likes(a,x)\ )$

y likes puffins:

$\forall x\ (\ Puffin(x) \rightarrow Likes(y,x)\ )$

Everyone likes puffins:

$\forall y\ \forall x\ (\ Puffin(x) \rightarrow Likes(y,x)\ )$

7. Quantifiers Bind Variables

Reading: §9.3

"If everything is square, everything is blue"
$\forall \mathbf{x}\ \mathbf{Square(x)} \rightarrow \forall \mathbf{x}\ \mathbf{Blue(x)}$

This quantifier binds this variable

Typically, a quantifier $\forall x$ or $\exists x$ **binds** all instances of the variable x in it's scope
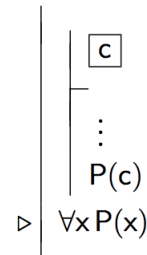
8. Summary of Quantifier Rules So Far

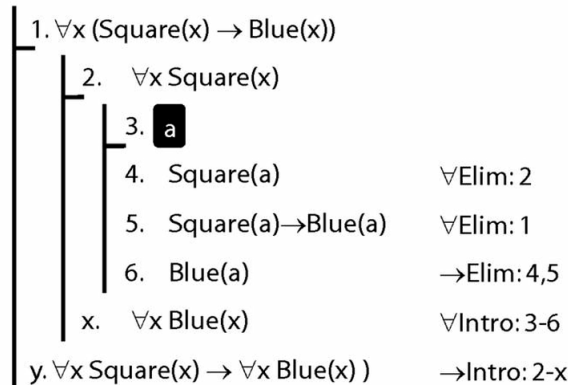*Reading:* §12.1, §12.2, §12.3, §13.1, §13.2
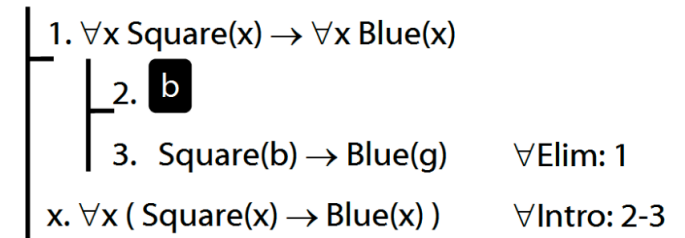
9. ∀Intro

Reading: §12.1, §12.3, §13.1

Universal Introduction (∀ Intro)

$$\boxed{c}$$
$$\vdots$$
$$P(c)$$
$$\triangleright\quad \forall x\, P(x)$$

where **c** does not occur outside the subproof where it is introduced.

1. $\forall x\ (Square(x) \rightarrow Blue(x))$
2. $\quad \forall x\ Square(x)$
3. $\quad \boxed{a}$
4. $\quad\quad Square(a)$ ∀Elim: 2
5. $\quad\quad Square(a) \rightarrow Blue(a)$ ∀Elim: 1
6. $\quad\quad Blue(a)$ →Elim: 4,5
x. $\quad \forall x\ Blue(x)$ ∀Intro: 3-6
y. $\forall x\ Square(x) \rightarrow \forall x\ Blue(x)\ )$ →Intro: 2-x

Why is this proof incorrect?

1. $\forall x\ Square(x) \rightarrow \forall x\ Blue(x)$
2. $\boxed{b}$
3. $\quad Square(b) \rightarrow Blue(g)$ ∀Elim: 1
x. $\forall x\ (\ Square(x) \rightarrow Blue(x)\ )$ ∀Intro: 2-3

10. ∀Intro: An Incorrect Proof

Reading: §13.1, §13.2

This proof is wrong, but why?:

1. $\forall x\ Square(x) \rightarrow \forall x\ Blue(x)$
2. $\boxed{b}$
3. $\quad Square(b) \rightarrow Blue(b)$ ∀Elim: 1
x. $\forall x\ (\ Square(x) \rightarrow Blue(x)\ )$ ∀Intro: 2-3
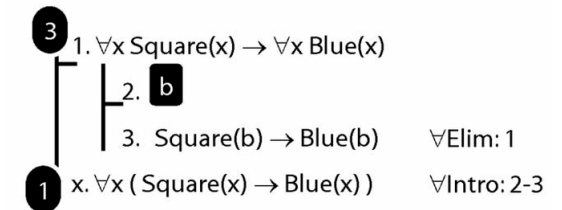
There is a counterexample to the argument:

1 is FALSE

3 is TRUE