Logic I: Lecture 12

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. Relations: Reflexive, Symmetric

Reading: §15.1

A *reflexive* relation is one that everything bears to itself. (E.g. everything is the SameShape as itself. E.g. of non-reflexive: not everything is LeftOf itself).

A *symmetric* relation is one such that if x bears it to y, then y bears it to x. (E.g. Adjacent(x,y) is symmetric, LeftOf(x,y) is not symmetric.)

2. Relations: Transitivity

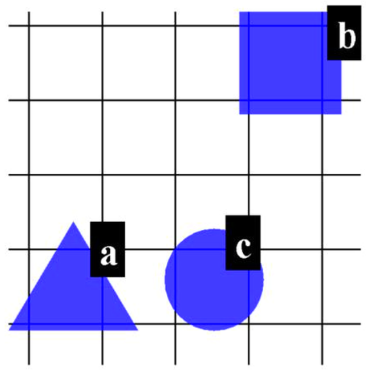
Reading: §15.1

A *transitive* relation is one such that if x bears it to y and y bears it to z then x bears it to z. (E.g. LeftOf is transitive; NotAdjacent is not transitive.)

If NotAdjacent were transitive, the following argument would be logically valid:

1. NotAdjacent(a, b)
2. NotAdjacent(b, c)
3. NotAdjacent(a, c)

A counterexample to this argument:



3. Relations: Some Examples

Reading: §15.1

	Reflexive	Symmetric	Transitive
NotEqual	N	Y	N
=	Y	Y	Y
SameShape			
DifferentShape			
LeftOf			
Adjacent	N	Y	N

Artificial relations …

EqualToOrLeftOf(x, y) iff

x = y or LeftOf(x, y)

EqualToOrAdjacent(x, y) iff

x=y or Adjacent(x, y)

JohnOrAyesha(x, y) iff

x = John and y = Ayesha

or x = Ayesha and y = John

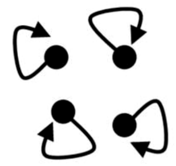
JohnToAyesha(x, y) iff

x = John and y = Ayesha

4. Expressing Relations with Quantifiers

Reading: §15.1

A *reflexive* relation is one that everything bears to itself. (E.g. SameShape)
reflexive: ∀x R(x,x)



A *symmetric* relation is one such that if x bears it to y, then y bears it to x. (E.g. Adjacent(x,y))
symmetric: ∀x∀y (R(x,y) → R(y,x))

A *transitive* relation is one such that if x bears it to y and y bears it to z then x bears it to z. (E.g. LeftOf is transitive; DifferentShape is not transitive)
transitive: ∀x∀y∀z ((R(x,y) ∧ R(y,z)) ⟶ R(x,z))

5. Negating Identity

¬a=b
¬(a=b)
} these mean the same thing

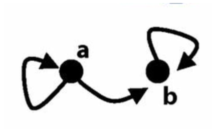
6. Expressing Counterexamples Formally

Reading: §15.1

Give a counterexample to this argument:

∀x R(x,x)
∀x∀y∀z ([R(x,y) ∧ R(y,z)] → R(x,z))
∀x∀y [R(x,y) → R(y,x)]

Informally:



Formally:

Domain: {a, b}

R: {<a,a>, <a,b>, <b,b>}

7. Proof Example: A∧B therefore ¬(¬A∨¬B).

1. A ∧ B

¬(¬A ∨ ¬B)

8. Proof Example: P therefore ¬¬P.

1. P

5. ¬¬P

9. Proof Example: S∨(Q∧R) therefore S∨Q.

1. S∨(Q∧R)

S∨Q