Logic I: Lecture 16

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Readings refer to sections of the course textbook, *Language, Proof and Logic*.

1. There Is Exactly One

There is one creator (at least one, maybe more).

$\exists x\ \mathrm{Creator}(x)$

Ahura Mazda is the one and only creator.

$\mathrm{Creator}(a) \wedge \forall x(\ \mathrm{Creator}(x) \rightarrow x{=}a\ )$

All squares are broken.

$\forall x(\ \mathrm{Sqr}(x) \rightarrow \mathrm{Brkn}(x)\ )$

There is one and only one creator.

$\exists y(\ \mathrm{Creator}(y) \wedge \forall x(\ \mathrm{Creator}(x) \rightarrow x{=}y\ )\ )$

or:

$\exists y\ \forall x(\ \mathrm{Creator}(x) \leftrightarrow x{=}y\ )$

2. Every Time I Go to the Dentist Someone Dies

Reading: §11.2

$\forall t\ ($

$(\ \mathrm{Time}(t) \wedge \mathrm{ToDentist}(a,t)\ )$

$\rightarrow$

$\exists x\ (\ \mathrm{Person}(x) \wedge \mathrm{TimeOfDeath}(x,t)\ )$

)

3. Could There Be Nothing?

Reading: §13.2

1.	a=a	=Intro
2.	∃x (x=x)	∃ Intro: 1

1.	...	
...		
m.	∀x (Train(x) ∨ ¬Train(x))	...
n.	Train(a) ∨ ¬Train(a)	∀Elim: m
o.	a=a	=Intro
p.	∃x (x=x)	∃ Intro: 1

	1. ¬(a=a)	
	2. a=a	=Intro
	3. ⊥	⊥Intro: 1,3
4.	¬¬(a=a)	¬Intro: 1-3
5.	a=a	¬Elim: 5
6.	∃x (x=x)	∃ Intro: 5

4. Proofs about Proofs

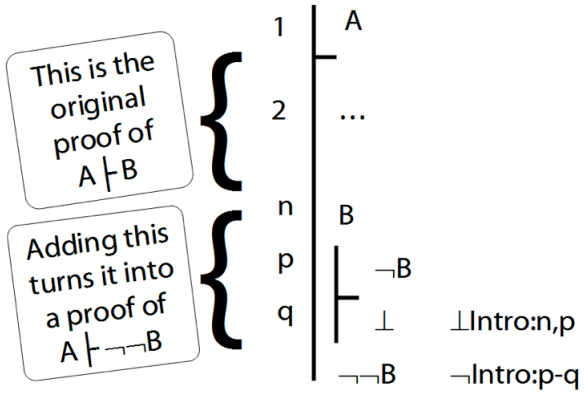
If A ⊢ B then ⊢ A⟶B
Proof Given a proof for A ⊢ B ...

1		A
		...
n		B

... we can turn it into a proof for ⊢ A⟶B:

1		A	
		...	
n		B	
	A→B		→Intro: 1-n

If $A \vdash B$ then $A \vdash \neg\neg B$
Proof:



If $A \vdash C$ then $A \vdash B \rightarrow C$

If $A \vdash B$ and $A \vdash \neg C$ then $A \vdash \neg(B \rightarrow C)$

5. Does 'if' mean what '→' means?

Reading: §7.3

These two arguments are valid: does that mean that 'if' means what '→' means?

$\neg A \vee B$	America does not exist ∨ Baudrillard is wrong
If A, B	If America exists, Baudrillard is wrong

If A, B	If you love logic, things will fall into place
$\neg(A \wedge \neg B)$	Not both: you take logic and things don't fall into place

The English argument isn't valid; the awFOL argument is valid; therefore 'if' can't mean what '→' means?

$\neg A$	Marnie will not miss her train
$A \rightarrow B$	If Marnie misses her train, she will arrive on time.