Logic I: Lecture 17

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Readings refer to sections of the course textbook, *Language, Proof and Logic.*

1. Revison: Definitions

Exercise State the rules of proof for the following two connectives: $\wedge$ $\longrightarrow$

What is a logically valid argument?

What is ... logical consequence, a tautology, a contradiction, a counterexample, a subproof, ...

What is a proof?

2. Revison: Truth tables

Use truth tables to establish whether the following arguments are valid. If any arguments are invalid, state counterexamples to them. If any arguments are valid, explain carefully using the truth tables why they are valid.

1.

$$\begin{array}{r|l} 1 & P \rightarrow Q \\ \hline 2 & \neg P \vee Q \end{array}$$

2.

$$\begin{array}{r|l} 1 & P \leftrightarrow (Q \rightarrow Q) \\ \hline 2 & P \vee Q \end{array}$$

3.

$$\begin{array}{r|l} 1 & P \vee \neg(Q \wedge R) \\ \hline 2 & P \vee (\neg Q \wedge R) \end{array}$$

3. Revison: Proofs (propositional)

1.

$$\begin{array}{r|l} 1 & \neg P \wedge R \\ \hline 2 & \neg P \end{array}$$

2.

$$\begin{array}{r|l} 1 & \neg P \vee R \\ \hline 2 & P \rightarrow R \end{array}$$

4. Revison: Proofs (with quantifiers)

1.

$$\begin{array}{r|l} 1 & \forall x S(x) \\ \hline 2 & \forall x \neg S(x) \\ \hline 3 & \bot \end{array}$$

2.

$$\begin{array}{r|l} 1 & \forall x (F(x) \longrightarrow x = a) \\ \hline 2 & \neg \exists x (F(x) \wedge \neg x = a) \end{array}$$

3.

$$\begin{array}{r|l} 1 & \exists x \forall y (F(y) \longrightarrow \neg G(x, y)) \\ \hline 2 & \forall y \exists x (F(y) \longrightarrow \neg G(x, y)) \end{array}$$

5. Revison: Translation from English to FOL

Exercise Translate the following sentences of English into FOL using the interpretation below:

L(x,y) : x is a logical consequence of y

N(x,y) : x is the negation of y

S(x) : x is a sentence

a : 'Fire melts ice'

i. 'Fire melts ice' is a sentence

ii. There is a sentence

iii. There is a sentence which is the negation of 'Fire melts ice'

iv. Some sentences are contradictions and all contradictions are logically equivalent.

6. Does 'if' mean what '$\longrightarrow$' means?

Reading: §7.3

These two arguments are valid: does that mean that ‘if’ means what ‘⟶’ means?

$\neg A \lor B$	America does not exist $\lor$ Baudrillard is wrong
If A, B	If America exists, Baudrillard is wrong

If A, B	If you love logic, things will fall into place
$\neg(A \land \neg B)$	Not both: you take logic and things don’t fall into

The English argument isn’t valid; the FOL argument is valid; therefore ‘if’ can’t mean what ‘⟶’ means?

$\neg A$	Marnie will not miss her train
$A \to B$	If Marnie misses her train, she will arrive on time.

7. Proof Example: $\exists x$ Dead(x) $\vdash \neg\forall x\neg$ Dead(x).

1.	$\exists x\, F(x)$	
2.	a $F(a)$	
3.	$\forall y \neg F(y)$	
4.	$\neg F(a)$	$\forall$Elim: 3
5.	$\bot$	$\bot$Intro: 2,4
6.	$\neg\forall y \neg F(y)$	$\neg$Intro: 3-5
7.	$\neg\forall y \neg F(y)$	$\exists$Elim: 2-6

8. Proof Example: $\neg\forall x$ Dead(x) $\vdash \exists x\neg$ Dead(x).

1.	$\neg\forall x$ Dead(x)	
2.	$\neg\exists x\neg$Dead(x)	
3.	a	
4.	$\neg F(a)$	
5.	$\exists x\neg$Dead(x)	$\exists$Intro: 4
6.	$\bot$	$\bot$Intro: 2,5
7.	$\neg\neg F(a)$	$\neg$Intro: 4-6
8.	$F(a)$	$\neg$Elim: 8
9.	$\forall x$ Dead(x)	$\forall$Intro: 3-8
10.	$\bot$	$\bot$Intro: 1,9
11.	$\neg\neg\exists x\neg$Dead(x)	$\neg$Intro: 2-10
12.	$\exists x\neg$Dead(x)	$\neg$Elim: 11

9. The End Is Near

Reading: §14.3

‘The’ can be a quantifier, e.g. ‘the square is broken’. How to formalise it?

The square is broken

$\Dashv\vDash$ There is exactly one square and it is broken

$\Dashv\vDash$ There is at most one square and there is at least one square and it is broken

$\Dashv\vDash$ There is at most one square and there is at least one square and all squares are broken

$\Dashv\vDash \neg \exists x \exists y$ (Square(x) $\land$ Square(y) $\land \neg$x=y)

$\land \exists x$ Square(x)

$\land \forall x$ (Square(x) $\longrightarrow$ Broken(x))

Which shorter sentences are equivalent to this?

$\exists x$ (Square(x) $\land \forall y$ (Square(y) $\longrightarrow$ y=x) $\land$ Broken(x))

$\exists x$ ($\forall y$ (Square(y) $\leftrightarrow$ y=x) $\land$ Broken(x))