Rules of Thumb for Logic 1

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There are exceptions to these rules of thumb. But they are often useful.

1. Proofs

1.1. Starting

First ask, 'Which $Elim$ rule can apply to this premise?' for each premise. Apply any $Elim$ rules you can first (except $\forall Elim$—see below).

Then ask, 'Which $Intro$ rule would get me to this conclusion?'

If you still can't get to the conclusion, try using $\neg Intro$. (You can do use $\neg Intro$ even if the conclusion isn't a negated sentence. For example, if the conclusion is $A \vee B$, create a subproof with $\neg(A \vee B)$ as premise, derive a contradiction, use $\neg Intro$ to get $\neg\neg(A \vee B)$ then use $\neg Elim$.)

1.2. $\forall Elim$

Use $\forall Elim$ as late as possible in your proof.

Only apply $\forall Elim$ using names that already occur in your proof.

1.3. $\perp$

Don't use $\perp Elim$: you need $\neg Intro$.

When using $\vee Elim$, if you are struggling to get two subproofs with matching conclusions try using $\perp Elim$ or $\vee Intro$.

1.4. What to do with $\neg$

Having sentences that start with negation ($\neg$) as premises is awkward. Learning some standard proofs will help you.

If you have $\neg(A \to B)$, you can get A like this:

1	$\neg(A \to B)$	
2	$\vert\ \neg A$	
3	$\vert\ \vert\ A$	
4	$\vert\ \vert\ \perp$	$\perp Intro$: 2,3
5	$\vert\ \vert\ B$	$\perp Elim$: 4
6	$\vert\ A \to B$	$\to Intro$: 3–5
7	$\vert\ \perp$	$\perp Intro$: 1, 7
8	$\neg\neg A$	$\neg Intro$: 2–7
9	A	$\neg Elim$: 8

If you have $\neg(A \to B)$, you can get $\neg B$ like this:

1	$\neg(A \to B)$	
2	$\vert\ B$	
3	$\vert\ \vert\ A$	
4	$\vert\ \vert\ B$	$Reit$: 2
5	$\vert\ A \to B$	$\to Intro$: 3–4
6	$\vert\ \perp$	$\perp Intro$: 1, 7
7	$\neg B$	$\neg Intro$: 2–6

If you have $\neg(A \vee B)$, you can get $\neg A$ like this:

1	$\neg(A \vee B)$	
2	$\vert\ A$	
3	$\vert\ (A \vee B)$	$\vee Intro$: 2
4	$\vert\ \perp$	$\perp Intro$: 1, 3
5	$\neg A$	$\neg Intro$: 2–4

You can use $\neg\exists x Blue(x)$ almost as if it were $\forall x \neg Blue(x)$: you can get $\neg Blue(b)$ like this:

1	$\neg\exists x Blue(x)$	
2	$\vert\ Blue(b)$	
3	$\vert\ \exists x Blue(x)$	$\exists Intro$: 2
4	$\vert\ \perp$	$\perp Intro$: 1, 3
5	$\neg Blue(b)$	$\neg Intro$: 2–4

2. Translation

Use $\forall$ with $\to$, e.g.

$$\forall x(Square(x) \to Broken(x))$$

means all squares are broken.

Use $\exists$ with $\wedge$, e.g.

$$\exists x(Square(x) \wedge Broken(x))$$

means some square is broken.

English sentences with mixed quantifiers are ambiguous (e.g. 'There is a store for everything.').